Invented (Personal) Strategies for Subtraction

as explained in Van de Walle,
<u>Teaching Student Centered Mathematics, grades 3-5,</u>
pages 110-112.

FIGURE 4.5

Subtraction by counting up is a powerful method.

Invented Strategies for Subtraction by Counting Up

Add Tens to Get Close, Then Ones

73 – 46

46 and 20 is 66.
(30 more is too much.)
Then 4 more is 70 and 3 is 73.
That's 20 and 7 or 27.

46 > 20
66 > 4
70 > 3
73
27

Add Tens to Overshoot, Then Come Back

73 – 46

46 and 30 is 76.
That's 3 too much, so it's 27.

73 – 46
46 + 30 → 76 – 3 → 73
30 – 3 = 27

Add Ones to Make a Ten, Then Tens and Ones

73 – 46

46 and 4 is 50.
50 and 20 is 70 and 3 more is 73. The 4 and 3 is 7 and 20 is 27.

73 – 46
46 + 4 → 50
+ 20 → 70
+ 3 → 73
27

Similarly,
46 and 4 is 50.
50 and 23 is 73.
23 and 4 is 27.

46 + 4 → 50
50 + 23 → 73
23 + 4 = 27

Invented Strategies for Take-Away Subtraction

Take Tens from the Tens, Then Subtract Ones

73 – 46

70 minus 40 is 30.
Take away 6 more is 24.
Now add in the 3 ones — 27.

73 – 46
70 – 40 → 30 – 6 →
24 + 3 → 27

Or

70 minus 40 is 30.
I can take those 3 away, but I need 3 more from the 30 to make 27.

~~73~~
– 46
30
– 3
27

Take Away Tens, Then Ones

73 – 46

73 minus 40 is 33.
Then take away 6:
3 makes 30 and 3 more is 27.

73 – 40 → 33 – 3
30 – 3 → 27

Take Extra Tens, Then Add Back

73 – 46

73 take away 50 is 23.
That's 4 too many.
23 and 4 is 27.

73 – 50 → 23 + 4
27

Add to the Whole If Necessary

73 – 46

Give 3 to 73 to make 76.
76 take away 46 is 30.
Now give 3 back — 27.

+3
73 – 46
76 – 46 → 30
– 3 → 27

FIGURE 4.6

Take-away strategies work reasonably well for two-digit problems. They are a bit more difficult with three digits.

Traditional Algorithms for Addition and Subtraction

The traditional computational methods for addition and subtraction are significantly different from nearly every invented method. In addition to starting with the rightmost digits and being digit oriented (as already noted), the traditional approaches involve the concept generally referred to as *regrouping,* exchanging 10 in one place-value position for 1 in the position to the left ("carrying"), or the reverse, exchanging 1 for 10 in the position to the right ("borrowing"). The terms *borrowing* and *carrying* are obsolete and conceptually misleading. The word *regroup* also offers no conceptual help to students. A preferable term is *trade*. Ten ones are *traded* for a ten. A hundred is *traded* for 10 tens. Trading makes sense with the use of base-ten pieces when, in fact, pieces must be traded; for example, a ten piece is traded in for 10 ones pieces.

Terminology aside, the trading process is quite different from the bridging process used in all invented and mental strategies. Consider the task of adding 28 + 65. Using the traditional method, we first add 8 and 5. The resulting 13 ones are separated into 3 ones and 1 ten. The newly formed ten is then combined with the other tens. This process of "carrying a ten" is conceptually difficult and is different from the bridging process that occurs in invented strategies. In fact, nearly all major textbooks now teach this process of regrouping prior to and separate from direct instruction with the addition and subtraction algorithm, an indication of the difficulties involved. The process is even more difficult for subtraction, especially across a zero in the tens place where two successive trades are required.

Compounding all of this is the issue of recording each step. The traditional algorithms do not lend themselves to mental computation, so students must learn to record. The literature of the past 50 years is replete with the errors that students make with these recording methods.

All of these observations are offered to encourage you to abandon the traditional algorithms for addition and subtraction and, failing that, to alert you to the difficulties