Strategies for Whole-Number Computation

Invented Strategies for Addition and Subtraction

Research has demonstrated that children will invent a lot of different strategies for addition and subtraction. Your goal might be that each of your students has at least one or two methods that are reasonably efficient, mathematically correct, and useful with lots of different numbers. Expect different children to settle on different strategies.

It is not at all unreasonable for students to be able to add and subtract two-digit numbers mentally in the third grade. However, even in fourth grade, do not push all students to pure mental computation. By recording on the board the ideas that students use, you help all students develop new approaches. Those who need short-term memory assistance can see ways to support their strategies by jotting down intermediate results on paper. The goal should be flexible, meaningful computation. These methods tend to become mental with frequent use.

Most of the ideas suggested here for addition and subtraction can be taught and even mastered by the end of second grade. However, most third-grade students and even fifth- and sixth-grade students have not developed invented strategies. The sequence of ideas proposed is appropriate at any grade.

Adding and Subtracting Single Digits

Children can easily extend addition and subtraction facts to higher decades.

Tommy was on page 47 of his book. Then he read 8 more pages. How many pages did Tommy read in all?

If students are simply counting on by ones, the following activity may be useful. It is an extension of the make-ten strategy for addition facts.

ACTIVITY 4.1

Ten-Frame Adding and Subtracting

Quickly review the make-ten idea from addition facts using two ten-frames. (Add on to get up to ten and then add the rest.) Challenge students to use the same idea to add on to a two-digit number as shown in Figure 4.3. Two students can work together. First, they make a specified two-digit number with the little ten-frame cards. They then stack up all of the less-than-ten cards and turn them over one at a time. Together they talk about how to get the total quickly.

The same approach is used for subtraction. For instance, for 53 – 7, take off 3 to get to 50, then 4 more is 46.

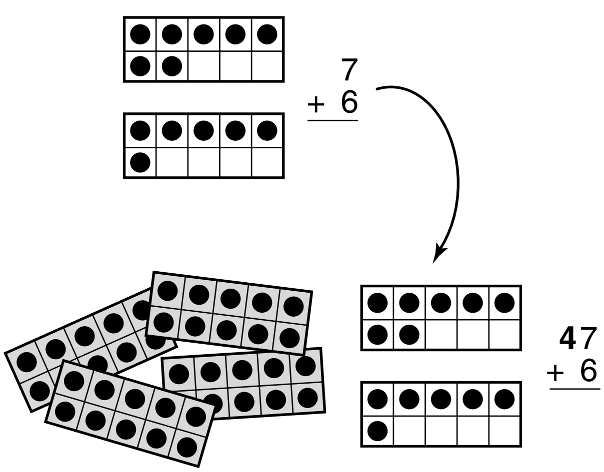

FIGURE 4.3

Little ten-frame cards can help students extend the make-ten idea to larger numbers.

Notice how building up through ten (as in 47 + 6) or down through ten (as in 53 – 7) is different from carrying and borrowing. No ones are exchanged for a ten nor a ten for ones. The ten-frame cards encourage students to work with multiples of ten without regrouping.

Another important model to use is the hundreds chart. The hundreds chart has the same tens structure as the little ten-frame cards. For 47 + 6 you count 3 to get out to 50 at the end of the row and then 3 more in the next row.

Adding Two-Digit Numbers

For each of the examples that follow, a possible recording method is offered. These are intended to be suggestions, not prescriptions. Students have difficulty inventing recording techniques. If you record their ideas on the board as they explain their ideas, you are helping them develop written techniques. You may even discuss recording methods with individuals or with the class to decide on a form that seems to work well. Horizontal formats encourage students to think in terms of numbers instead of digits. A horizontal format is also less likely to encourage use of the traditional algorithms.

Students will often use a counting-by-tens-and-ones technique for some of these methods. That is, instead of "46 + 30 is 76," they may count "46 ⟶ 56, 66, 76." These counts can be written down as they are said to help students keep track.

Figure 4.4 illustrates four different strategies for addition of two two-digit numbers. The following story problem is a suggestion.

The two Scout troops went on a field trip. There were 46 Girl Scouts and 38 Boy Scouts. How many Scouts went on the trip?

The *move to make-ten* and *compensation* strategies are useful when one of the numbers ends in 8 or 9. To promote that strategy, present problems with addends like 39 or 58. Note that it is only necessary to adjust one of the two numbers.

STOP **Try adding 367 + 155 in as many different ways as you can. How many of your ways are like those in Figure 4.4?**

Invented Strategies for Addition with Two-Digit Numbers

Add Tens, Add Ones, Then Combine

46 + 38

40 and 30 is 70.
6 and 8 is 14.
70 and 14 is 84.

46
+38
70
14
84

Move Some to Make Tens

46 + 38

Take 2 from the 46 and put it with the 38 to make 40. Now you have 44 and 40 more is 84.

2
~~46~~ + ~~38~~
44 + 40
84

Add On Tens, Then Add Ones

46 + 38

46 and 30 more is 76.
Then I added on the other 8.
76 and 4 is 80 and 4 is 84.

46 + 38 →
76 + 8 → 80, 84

Use a Nice Number and Compensate

46 + 38

46 and 40 is 86.
That's 2 extra, so it's 84.

46 + 38
46 + 40 →
86 − 2 → 84

FIGURE 4.4 Four different invented strategies for adding two two-digit numbers.

Subtracting by Counting Up

This is an amazingly powerful way to subtract. Students working on the *think-addition* strategy for their basic facts can also be solving problems with larger numbers. The concept is the same. It is important to use *join with change unknown* problems or *missing-part* problems to encourage the counting-up strategy. Here is an example of each.

Sam had 46 baseball cards. He went to a card show and got some more cards for his collection. Now he has 73 cards. How many cards did Sam buy at the card show?

Juanita counted all of her crayons. Some were broken and some not. She had 73 crayons in all. 46 crayons were not broken. How many were broken?

The numbers in these problems are used in the strategies illustrated in Figure 4.5.

Emphasize the value of using tens by posing problems involving multiples of 10. In 50 – 17, the use of ten can happen by adding up from 17 to 20, or by adding 30 to 17. Some students may reason that it must be 30-something because 30 and 17 is less than 50, and 40 and 17 is more than 50. Because it takes 3 to go with 7 to make 10, the answer must be 33. Work on naming the missing part of 50 or 100 is also valuable. (See Activity 2.18, "The Other Part of 100," p. 54.)

Take-Away Subtraction

Take-away methods are more difficult to do mentally or even with the help of paper and pencil. This is especially true when problems involve three digits. Exceptions involve problems such as 423 – 8 or 576 – 300 (subtracting a number less than 10 or a multiple of 10 or 100). However, take-away strategies are bound to occur, probably because traditional textbooks emphasize take-away as the meaning of subtraction. Take-

FIGURE 4.5

Subtraction by counting up is a powerful method.